WS 81

1) $x - y = 50$ $\quad P(x,y) = xy$

$x = 50 + y$ $\quad P(y) = (50+y)y$

$P(y) = 50y + y^2$

$P'(y) = 50 + 2y = 0$

$y = -25$

$x = 25$

$P''(y) = 2 > 0$

Since $P'(-25) = 0$ & $P''(-25) > 0$

$P(y)$ is minimized when $y = -25$

The product of $xy = -625$.

2) $x + y = 36$ $\quad P(x,y) = xy$

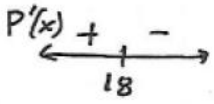
$y = 36 - x$ $\quad P(x) = x(36-x)$

$= 36x - x^2$

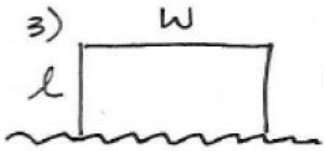
$P'(x) = 36 - 2x = 0$

$x = 18$

$y = 18$



P is maximized when $x = 18$ & $y = 18$ b/c $P'(x)$ Δs signs from + to − @ $x = 18$.

3) 

$P = 100$

$2l + w = 100$

$w = 100 - 2l$

$A(l,w) = l \cdot w$

$A(l) = l(100 - 2l)$

$= 100l - 2l^2$

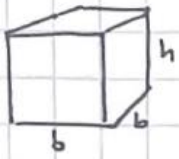
$A'(l) = 100 - 4l = 0$

$l = 25$

$w = 50$

$A''(l) = -4 < 0$

The plot of land is max when $l = 25$ ft & $w = 25$ ft.

4) 

$S = 2b^2 + 4bh = 600$

$4bh = 600 - 2b^2$

$h = 75b^{-1} - \frac{1}{2}b$

$V = b^2 h$

$V = b^2(75b^{-1} - \frac{1}{2}b)$

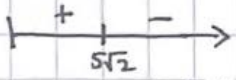
$V = 75b - \frac{1}{2}b^3$

$V' = 75 - \frac{3}{2}b^2 = 0$

$\frac{3}{2}b^2 = 75$

$b^2 = 50$

$b = 5\sqrt{2}$



$V = 75(5\sqrt{2}) - \frac{1}{2}(5\sqrt{2})^3$

$V = 250\sqrt{2}\ ft^3$

5)

$y = 2x^3 - 3x^2 - 12x$ on $[-2, 3]$

$y' = 6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad x = -1$

x	y
-2	-4
-1	7
2	-20
3	-9

y has abs max of $(-1, 7)$.

y has abs min @ $(2, -20)$.

6)

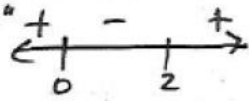
$y = x^4 - 4x^3$

$y' = 4x^3 - 12x^2$

$y'' = 12x^2 - 24x = 0$

$12x(x-2) = 0$

$x = 0 \quad x = 2$



$y(0) = 0$

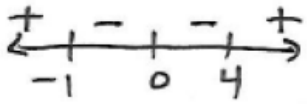
$y(2) = -16$

y has P.o.I @ $(0,0)$ & $(2,-16)$ b/c y'' Δs signs.

7)

$f'(x) = x^2(x+1)^3(x-4)^3 = 0$

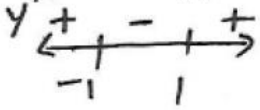
$x = 0 \quad x = -1 \quad x = 4$



8) $y = x^3 - 3x + 1$

$y' = 3x^2 - 3 = 0$

$x = \pm 1$



9)

a) $g(x) = \int_1^x f(t)\,dt \quad g(3) = \int_1^3 f(t)\,dt = 1 + \frac{1}{2}(1)(3) + \frac{1}{2}(1)(1) = 3$

$g'(x) = f(x) \quad g'(3) = f(3) = 0$

$g''(x) = f'(x) \quad g''(3) = f'(3) = -1$

b) g is inc on $(-2, 3)$ b/c $g'(x) > 0$

c) $g(-2) = \int_1^{-2} f(t)\,dt = -\frac{1}{2}(3)(4) = -6$

$g(3) = 3$

$g(4) = 3 - \frac{1}{2} = \frac{5}{2}$

} check endpoints & critical points

d) g has a P.o.I @ $x = 1$ b/c g'' Δs signs.

10)

$$F(x) = \int_0^{x^3} (2t-1)^2 dt$$

$$F'(x) = (2x^3-1)^2 \cdot 3x^2$$

11)

$$y = \frac{x^2+x}{x^2-7x+8} = \frac{x(x+1)}{(x-8)(x+1)}$$

y has a removable disc. @ $x=-1$

y has an infinite disc @ $x=8$.